RODOLFO GUIDOBONO | CURSO TEORÍA CUÁNTICA DE CAMPOS EJERCICIO CAPÍTULO 68 | JAVIER GARCÍA

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EJERCICIO (51:50)

Calcular $\langle \vec{k_1}\,\vec{k_2} \mid \vec{q_1}\,\vec{q_2} \rangle$

Considerar que:

[1] $a_{(\vec{q_1})}{}^{\dagger}|0\rangle = |\vec{q_1}\rangle$

[2] $a_{(\vec{q_1})}|0\rangle = 0$

[3] $\left[a_{(\vec{k})}\,,a_{(\vec{p})}{}^{\dagger}\right] = (2\pi)^3\delta^{(3)}{}_{(\vec{k}-\vec{p})}$

[4] $\langle 0 \mid 0 \rangle = 1$

$|\vec{q_1}\,\vec{q_2}\rangle = a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle$

$|\vec{k_1}\,\vec{k_2}\rangle = a_{(\vec{k_1})}{}^{\dagger}a_{(\vec{k_2})}{}^{\dagger}\,|0\rangle$

$\langle\vec{k_1}\,\vec{k_2}| = \langle 0|\,a_{(\vec{k_2})}a_{(\vec{k_1})}$

$\langle \vec{k_1}\,\vec{k_2} \mid \vec{q_1}\,\vec{q_2} \rangle = \langle 0|\,a_{(\vec{k_2})}a_{(\vec{k_1})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle$

Por [3]:

$\langle \vec{k_1}\,\vec{k_2} \mid \vec{q_1}\,\vec{q_2} \rangle = \langle 0|\,a_{(\vec{k_2})}\left(a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})} + (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}\right)a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle$

$\langle \vec{k_1}\,\vec{k_2} \mid \vec{q_1}\,\vec{q_2} \rangle = \langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle + \langle 0|\,a_{(\vec{k_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle$

En el primer término, por [3]:

$\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = \langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}\left(a_{(\vec{q_2})}{}^{\dagger}a_{(\vec{k_1})} + (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}\right)|0\rangle$

$\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = \langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{q_2})}{}^{\dagger}a_{(\vec{k_1})}\,|0\rangle + \langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}\,|0\rangle$

Por [2]:

$\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = 0 + (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}\,|0\rangle$

$\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}\langle 0|\,\left(a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_2})} + (2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_1})}\right)|0\rangle$

Por [2] y [4]:

$\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_1})}{}^{\dagger}a_{(\vec{k_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_1})}$

En el segundo término, por [3]:

$\langle 0|\,a_{(\vec{k_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}\langle 0|\,a_{(\vec{k_2})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle$

$$\langle 0|\, a_{(\vec{k_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}\langle 0|\, a_{(\vec{q_2})}{}^{\dagger}a_{(\vec{k_2})} + (2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_2})}\,|0\rangle$$

Por [2] y [4]:

$$\langle 0|\, a_{(\vec{k_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}\left(\langle 0|\, a_{(\vec{q_2})}{}^{\dagger}a_{(\vec{k_2})}\,|0\rangle + \langle 0|\,(2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_2})}\,|0\rangle\right)$$

$$\langle 0|\, a_{(\vec{k_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}\left(0 + (2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_2})}\right)\langle 0\mid 0\rangle$$

$$\langle 0|\, a_{(\vec{k_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}a_{(\vec{q_2})}{}^{\dagger}\,|0\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_2})}$$

$$\left\langle \vec{k_1}\,\vec{k_2}\mid\vec{q_1}\,\vec{q_2}\right\rangle = (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_1})} + (2\pi)^3\delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}(2\pi)^3\delta^{(3)}{}_{(\vec{k_2}-\vec{q_2})}$$

$$\boxed{\boldsymbol{\left\langle \vec{k_1}\,\vec{k_2}\mid\vec{q_1}\,\vec{q_2}\right\rangle = (2\pi)^6\left(\delta^{(3)}{}_{(\vec{k_1}-\vec{q_2})}\delta^{(3)}{}_{(\vec{k_2}-\vec{q_1})} + \delta^{(3)}{}_{(\vec{k_1}-\vec{q_1})}\delta^{(3)}{}_{(\vec{k_2}-\vec{q_2})}\right)}}$$